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Engineering Notes

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Evaluation of Dynamic Burn Rate from Extinction Compliance of Solid Rocket Motors

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Introduction

THE modeling of solid propellant combustion serves a number of useful purposes in the scheme of propellant development. Composite propellant burn-rate modeling has developed to a point where it can and does make useful contributions to combustion research and practical propellant development [1–18]. The available models are able, at least qualitatively, to explain the burn-rate characteristics of a wide variety of propellants of interest. However, deficiencies do remain and there is work yet to be done to improve the quantitative aspects and predictive capability in general [15].

A modeling restriction that is common to most works is the quasi-steady gas phase and homogeneous solid, one-dimensional flame (QSHOD) model, which neglects the gas-phase thermal inertia. The validity of the QSHOD model is limited to pressure perturbations within a low frequency range of below 1 kHz [10]. An analysis considering finite gas-phase thermal inertia, as well as the effects of nonlinear stability becomes formidable, and numerical methods have to be adapted. In spite of some attempts, fully workable solutions are not available [10]. Examination of frequency response function reveals that the role of gas-phase thermal inertia is not only to stabilize the burning near the first resonant mode but also to destabilize the burning near the second resonant mode. Calculations made for different amplitudes of driving pressure show that the nonlinear effects could be important in the dynamic behavior of burn rate. These are succinctly reported by Anil Kumar and Lakshmisha [10].

In the recent past, there has been a continued trend towards the development of theoretical models and new experimental techniques for the evaluation of burn rate. A judgement will not be made as to the best model or approach. There are many similarities and, where there are differences, each has its own virtues and faults. However, in the ballistician's point of view most of the theoretical models are too complicated for practical applications and the available experimental techniques are not assuring the prescribed accuracy and repeatability for the performance evaluation of solid rocket motors (SRM). Hence, it is necessary, even desirable, to formulate a simple mathematical model with less empirical constants. For that, it is inevitable to formulate an experimental scheme for getting unique values of empirical constants.

In this Note, an approximate analytical model has been developed to predict the dynamic burn rate of SRM from its "extinction compliance." The extinction compliance is defined in this Note as the ratio of the instantaneous chamber pressure and the steady-state burn rate. It can be evaluated from the depressurization test. It is well known that the depressurization test is somewhat unique and will yield repeatable experimental results with much less ambiguity and data scatter. The author believes that the unique extinction compliance will capture the physics and chemistry of the dynamic burning of the real motors, which will simplify the performance evaluation of real motors with less empiricism.

Description of the Model

The burn-rate law in the nonsteady form can be correlated with the pressurization rate ($\mathrm{d}P/\mathrm{d}t$) and may be written as

$$r(t) = \int_0^t \phi(t) \frac{\mathrm{d}P}{\mathrm{d}t}\,\mathrm{d}t \tag{1}$$

where $\phi(t)$ is defined as the "burn-rate kernel."

It is a formidable task to formulate the burn-rate kernel directly in a unique functional form, and so an alternate approach is adopted for modeling.

The burn-rate superposition principle here states that the sum of the burn-rate output resulting from each component of pressure input (in the form of pressure-rise rate) is the same as the burn-rate output resulting from the combined pressure input.

The burn rate can be represented mathematically under variable pressure as

$$r(t) = \sum_{i=1}^{m} \Delta P_i(t)\beta(t-\xi_i)H(t-\xi_i) \tag{2}$$

where $\beta(\mathrm{t})$ is the burn-rate compliance, $H(t)$ is the Heaviside function, m is the number of terms, i is the dummy subscript, t is time, and ξ is arbitrary time.

If the number of steps tends to infinity, the total burn rate can be expressed by an integral representation as

$$r(t) = \int_0^t \beta(t-\xi_i)H(t-\xi_i)\,\mathrm{d}[p(\xi)] \tag{3}$$

If the pressure history is differentiable, and because the dummy variable ξ is always less than or equal to t, the function $H(t-\xi)$ is therefore always unity in the range of integration. Hence, the preceding equation reduces to the following form:

$$r(t) = \int_0^t \beta(t-\xi) \frac{\partial P(\xi)}{\partial \xi}\,\mathrm{d}\xi \tag{4}$$

where $\mathrm{d}[P(\xi)]$ has been replaced by $[\partial P(\xi)/\partial \xi]\,\mathrm{d}\xi$ in order that time ξ may be the independent variable. Applying Laplace transform () yields the algebraic equation [Eq. (4)] in the transform variable s:

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$$\hat{r}(s) = s\hat{\beta}(s)\hat{P}(s) \tag{5}$$

Exactly the same arguments apply when the arbitrary changes in burn rate (extinction rate) are applied and the resulting change in pressure as a function of time is determined. Thus, the following equation will be obtained for pressure relaxation under arbitrarily prescribed burn rate, r:

$$P(t) = \int_0^t \varepsilon(t-\xi)\frac{\mathrm{d}r(\xi)}{\mathrm{d}\xi}\,\mathrm{d}\xi \tag{6}$$

Where $\varepsilon(t)$ is called "extinction compliance." The extinction compliance $[\varepsilon(t) = P(t)/r_o]$ can be evaluated from the depressurization test. From the depressurization test data, the constants can be evaluated through the conventional method of goodness of fit. It is important that Eqs. (2) and (6) must be obtained from the same system for balancing the physics and chemistry during the pressurization and depressurization processes.

Applying Laplace transform to Eq. (6) yields

$$\hat{P}(s) = s\hat{\varepsilon}(s)r(\hat{s}) \tag{7}$$

where s is the Laplace transform variable. Combining Eqs. (5) and (7), we get

$$\hat{\beta}(s)\hat{\varepsilon}(s) = \frac{1}{s^2} \tag{8}$$

This relation shows that one should be predictable if the other is known. It may be noted here that the extinction compliance of SRM can be found out more accurately than its burn-rate compliance from depressurization tests. It has already been reported by the earlier investigators from the scientific viewpoint that the depressurization test will yield repeatable experimental results with much less ambiguity and data scatter [4,5]. Hence, this proposed indirect approach is inevitable for getting the unique burn-rate compliance.

Solution Methodology

For illustration, a dynamic burn-rate model has been predicted from the extinction compliance $[\varepsilon(t) = P(t)/r_o]$ of a solid propellant rocket motor.

From the available depressurization test results, extinction compliance of an experimental solid rocket motor was fitted in the simplified form as

$$\varepsilon(t) = Ae^{-bt} \tag{9}$$

Note that one can also attempt to predict the theoretical values of A and b, where A is a function of P^n at steady-state condition and b is a function of propellant surface temperature and activation energy. The Laplace transform of Eq. (9) becomes

$$\hat{\varepsilon}(s) = A\left[\frac{1}{s+b}\right] \tag{10}$$

Substituting Eq. (10) into Eq. (8) one obtains

$$\hat{\beta}(s) = \frac{1}{A}\left[\frac{1}{s} + \frac{b}{s^2}\right] \tag{11}$$

Inserting Eq. (11) into Eq. (5), and solving for $r(s)$, yields

$$\hat{r}(s) = \frac{1}{A}\hat{P}(s) + \frac{b}{A}\frac{1}{s}\hat{P}(s) \tag{12}$$

The inverse Laplace transform of Eq. (12) yields the dynamic burn rate

$$r(t) = \frac{1}{A}P(t) + \frac{b}{A}\int_0^t P(\xi)\,\mathrm{d}\xi \tag{13}$$

where

$$P(t) = P_i + (t-t_i)\frac{\mathrm{d}P}{\mathrm{d}t} \tag{14}$$

Therefore, the dynamic burn rate can be expressed as

$$r(t) = \frac{1}{A}\left[P_i + (t-t_i)\frac{\mathrm{d}P}{\mathrm{d}t}\right] + \frac{b}{A}\int_{t_i}^{t_{i+1}}\left[P_i + (t-t_i)\frac{\mathrm{d}P}{\mathrm{d}\xi}\,\mathrm{d}\xi\right]\mathrm{d}t \tag{15}$$

Note that the constants A and b are the same as the constants that appeared in Eq. (9), which is represented as the best-fit curve of the extinction compliance. In general, from the available depressurization test data, functional constants can be evaluated through the conventional method of goodness of fit. It may be noted here that only for simple forms of extinction compliance is an analytical integration possible.

The proposed simple model presented, as Eq. (8) in transform variable s, is best regarded as a basis for modeling the dynamic burn rate of SRM, because the extinction compliance of a solid propellant rocket motor can be found out more accurately than the burn-rate compliance. Through this modeling, a novel relationship has been established between the burn-rate compliance and the extinction compliance. The methodology presented here is considered to be the most successful and simple one for predicting the dynamic burn rate of SRM because depressurization tests will offer great potential for formulating the unique functional relation of extinction compliance of solid rocket motors in general. Moreover, the proposed scheme is easier and more inexpensive than the available techniques.

In this paper, the Laplace transform has been effectively applied for transforming pressure-rise rate into dynamic burn rate. The pressure that varies with time in an irregular manner can be approximated either by intervals consisting of pressure steps (quasi steady) or by intervals of constant pressure-rise rate. In this approach, assumption of constant pressure-rise rate has been applied and hence it retains the integral form in the model. Obviously, this is more accurate than the conventional former one. Hence, it is made possible to formulate a simple analytical model, in terms of pressure and pressure-rise rate, for predicting the dynamic burn rate. Although many experimental depressurization data were available in open literature, they did not meet the needs of this model for validation. However, Suhas and Bose [4,5] were generous enough to furnish their detailed experimental results that enabled the author to make a direct comparison of the predicted and measured burn rate of composite propellants.

Review of Suhas and Bose's Depressurization Test

Suhas and Bose [4,5] carried out the depressurization test at their laboratory at the Aeronautical Engineering Department of the Indian Institute of Technology, Madras (IITM), Chennai and reported their complete results in 1980. They used ammonium perchlorate (AP)/Carboxyl-terminated polybutadiene (CTPB) and AP/Polybutadiene-acrylonitrile (PBAN) propellants strands for their in-house tests. These strands were allowed to burn in a closed chamber, in which pressure was rapidly reduced by rupturing a membrane at the wall along a nozzle, which controls the rate of pressurization.

Figure 1a shows the experimental setup for the depressurization study, which was developed by Suhas [4]. Typical test data and the measured time history of pressure are shown in Figs. 1b–1d, which are reproduced from [4]. The dynamic burn rate was measured by using an oscillator-discriminator-differentiator circuit. Suhas reported that the experimental results compare well with those of Yin and Hermance who have also used oscillator-discriminator-differentiator circuit for measuring the dynamic burn rate [9]. However, the difference between the circuits was that the centerline frequency of the discriminator used by Suhas [4] at IITM was 4.46 MHz whereas that used by Yin and Hermance [9] was 10.7 MHz. Also, Yin and Hermance had used a magnetic taps recorder to record the signals and not a storage oscilloscope as had been used at the IITM. Suhas [4] reported that the experiments could not be carried out at higher pressures nor at higher pressure decay

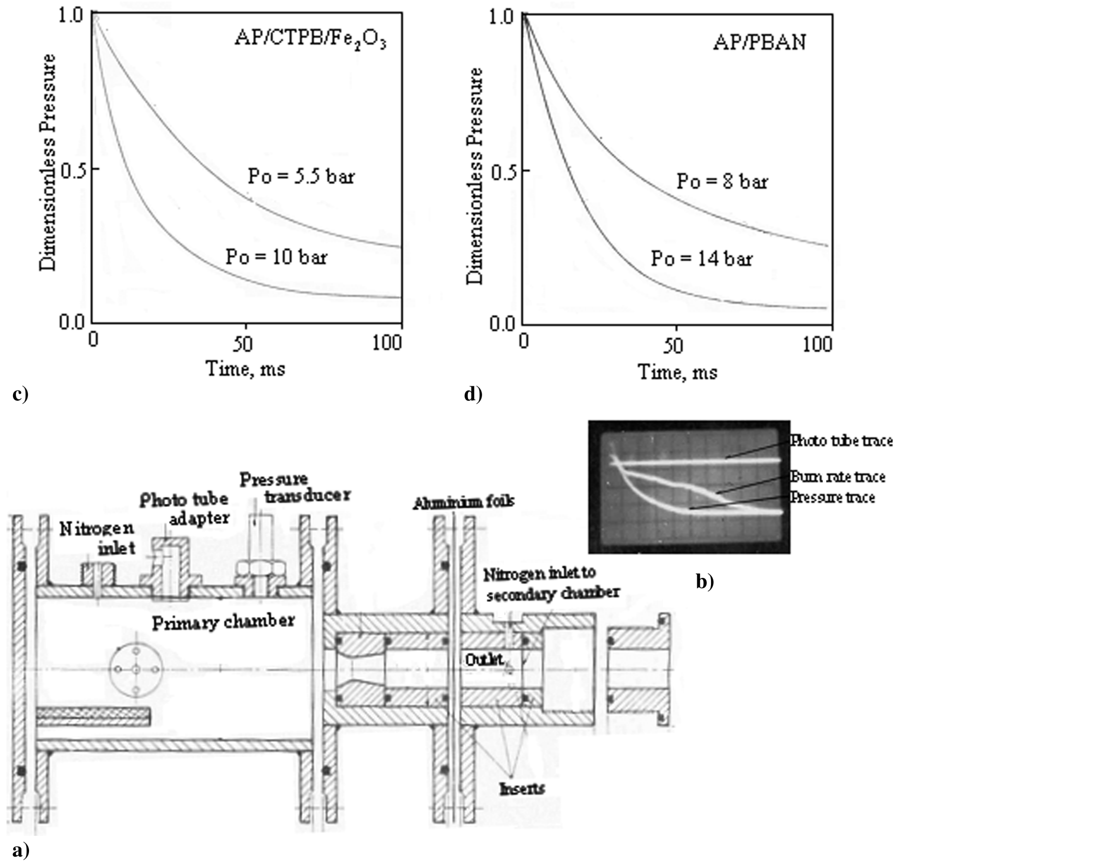

Fig. 1 a) Suhas' experimental setup for depressurization study, b) typical test results, and c), d) measured time history of pressure. (From Suhas [4].)

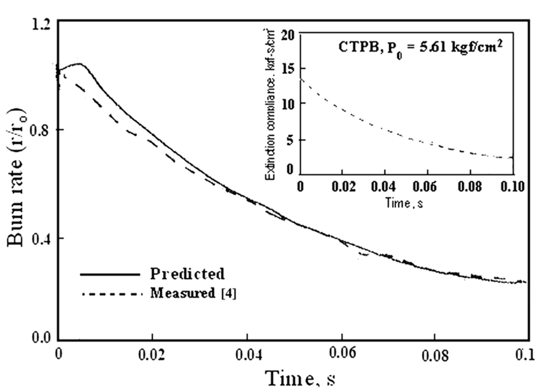

Fig. 2 Comparison of measured and predicted burn rate of CTPB-based propellant.

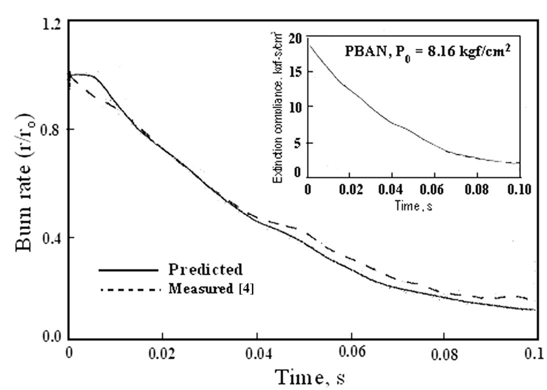

Fig. 3 Comparison of measured and predicted burn rate of PBAN-based propellant.

rates for the reason that the vibrations created due to the rupturing of the aluminum diaphragms resulted in a malfunction of the oscilloscope. Suhas described to the author that during the late 70s they had used a simple L-C circuit called an FM modulator to which a capacitor made out of propellant was connected.† The experiment was involved in recording the changing frequency (due to changing L-C combination) due to the burning of propellant (capacitor). Because it was using an L-C combination, it had a very limiting capability compared with microwave Doppler shift techniques used by U.S. laboratories at that time, due to the time scale of the circuit used to measure the change. Nevertheless, the measured dynamic burn-rate data and the corresponding depressurization history at different pressure levels (5–14 bars) reported in the Ph.D. dissertation of Suhas [4] are sufficient to validate the proposed dynamic burn-rate model.

†H. K. Suhas, personal communication, 06 October 2006.

Results and Discussion

Figures 2 and 3 show the comparison of the measured and the predicted burn rate of PBAN and CTPB-based propellants from the experimental results of Suhas and Bose [4,5]. The predicted history of the corresponding extinction compliance of these composite propellants is shown in the inset of Figs. 2 and 3 respectively. In all the cases considered, the extinction compliance is found to be exponential in nature. It was a fair approximation of the relationship encountered experimentally in the form of Eq. (9); and constant A and b appearing in this equation are evaluated as best-fit. Dynamic burn rate has been computed based on the model presented as Eq. (15). Pressure-time history during depressurization has been taken from [4] for computation. In general, theoretical predictions agreed well with both the experimental results gathered in this Note and those published by other investigators. The proposed technique is simple, inexpensive, and accurate for the prediction of dynamic burn rate of all class of solid propellant motors. Moreover, the results indicate that this burn-rate model is efficient and can be directly applied for the performance evaluation of solid rocket motors.

Conclusions

A novel mathematical technique has been developed to predict the dynamic burn rate of solid rocket motors from their experimentally determined extinction compliance. Using this technique, a powerful relation has been introduced in transform variable between the burn-rate compliance and the extinction compliance. This relation shows that one should be predictable if the other is known. Depressurization test results are used for formulating the extinction compliance through the method of goodness of fit. The Laplace transform has been effectively applied for transforming pressure-rise rate into dynamic burn rate. In this Note, assumption of constant pressure-rise rate has been applied and hence it retains the integral form in the model. Obviously, this is more accurate than the conventional quasi-steady assumption. Hence, it made possible to formulate a simple analytical model, in terms of pressure and pressurization rate, for predicting the dynamic burn rate. The theoretical predictions have been compared with the experimental results published by the earlier investigators. Excellent agreement has been observed and two cases are depicted in this paper. The proposed technique is simple, inexpensive, and efficient for the prediction of dynamic burn rate of all class of solid propellant rocket motors.

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